Guide to Transmission Line Constants Calculations (Impedance, Inductance and Capacitance)

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INTRODUCTION

Power system design always requires information of line constants and parameters such as impedance, inductance and capacitance. Those are particularly essential for relay engineers or power protection designers when they set a relay or design a protection scheme. However, incorrect information of line constants can sometimes wrongly affect the whole protection system or cause protection fails in power systems. In order to avoid any errors caused by missing line parameters or incorrect information, designers or engineers frequently need to calculate them. This calculation can offer sequence impedance value as well as phase impedance. The effectiveness of the calculation will be introduced in this report with an example of unbundled 3 phase and ground cable structure. The list of explanations for the line constants calculation is shown as below:

- Information of Conductors for calculation of Line Parameter
- Dimension at a Line Pole
- Calculation of Line Constants (Assumed No Ground Wire)
- Calculation of Line Constants (With Ground Wire)
- Line Constant Program Comparison

INFORMATION OF CONDUCTORS FOR CALCULATION OF LINE PARAMETER

Information of conductors should be ready first before line constants calculation is performed. The elements which are required for calculation are as follows:

- Diameter of phase conductor
- Diameter of ground wire
- Conductor's operating temperature
- Resistances of conductor by temperature (Ω/mile)
- Frequency of power system
- Distance between phase

If the conductor's type and producer are known, the information above could be easily found in producers' website. Most cable production companies offer their product's information similar to the table below. The distance between phases will be explained more specifically in the next paragraph.

ACSS/TW

Shaped Wire Concentric-Lay Compact Aluminum Conductors Steel Supported (ACSS/TW)
Diameter Equal to Standard ACSR Sizes

Code Word	Size (kcmil)	Type No.	Cross Sectional Area (in^2)		Stranding			Diameter (in)		Weight (lbs/1000 ft)			Rated Breaking Strength (lbs)			Resistance (ohms/mile)		Ampacity (amps)				
			Aluminum	Total	No of Layers of Aluminum	No. of Aluminum Wires	No. & Diameter Individual Steel Wire	Steel Core	Complete Cable	Alum.	Steel	Total	Standard Strength	High Strength	HS-285	DC @ 20°C	AC @ 75°C	@ 75°C	@ 100°C	@ 150°C	@ 200°C	@ 250°C
Mohawk/ACSS/TW	571.7	13	0.4490	0.5074	2	18	7 x 0.1030	0.3090	0.846	536.6	197.5	734.1	15,600	17,100	19,700	0.1527	0.1884	725	889	1121	1294	1441
Calumet/ACSS/TW	565.3	16	0.4439	0.5162	2	20	7 x 0.1146	0.3441	0.858	531.2	244.4	775.6	18,400	20,200	23,500	0.1540	0.1898	725	890	1122	1295	1442
Mystic/ACSS/TW	666.6	13	0.5236	0.5914	2	20	7 x 0.1111	0.3333	0.913	625.7	229.7	855.4	18,200	19,900	22,900	0.1310	0.1619	798	980	1238	1431	1595
Oswego/ACSS/TW	664.8	16	0.5221	0.6072	2	20	7 x 0.1244	0.3732	0.927	624.6	288.0	912.6	21,700	23,400	27,200	0.1309	0.1616	802	985	1244	1439	1604
Maumee/ACSS/TW	768.2	13	0.6034	0.6819	2	20	7 x 0.1195	0.3585	0.977	721.1	265.8	986.9	21,000	23,000	26,500	0.1137	0.1407	872	1072	1356	1569	1750
Wabash/ACSS/TW	762.8	16	0.5992	0.6966	2	20	7 x 0.1331	0.3993	0.990	716.7	329.7	1046	24,900	26,800	31,200	0.1141	0.1411	873	1074	1359	1573	1755
Kettle/ACSS/TW	957.2	7	0.7518	0.8038	3	32	7 x 0.0973	0.2919	1.060	901.6	176.2	1078	16,800	18,100	20,400	0.0922	0.1180	973	1197	1514	1753	1955
Fraser/ACSS/TW	946.7	10	0.7436	0.8168	3	35	7 x 0.1154	0.3462	1.077	892.6	247.9	1141	21,100	22,900	26,200	0.0930	0.1187	974	1199	1517	1756	1959
Columbia/ACSS/TW	966.2	13	0.7589	0.8573	2	21	7 x 0.1338	0.4014	1.092	906.9	333.2	1240	26,400	28,300	32,800	0.0904	0.1124	1005	1239	1571	1822	2035
Suwannee/ACSS/TW	959.6	16	0.7537	0.8762	2	22	7 x 0.1493	0.4479	1.108	901.6	414.9	1317	30,700	33,100	38,600	0.0907	0.1127	1008	1243	1576	1828	2042
Cheyenne/ACSS/TW	1168.1	5	0.9175	0.9646	3	30	7 x 0.0926	0.2778	1.155	1099	159.6	1259	17,200	18,300	20,500	0.0757	0.0979	1095	1350	1712	1986	2219
Genesee/ACSS/TW	1158.0	7	0.9095	0.9733	3	33	7 x 0.1078	0.3234	1.165	1091	216.3	1307	20,500	22,100	25,000	0.0762	0.0981	1094	1350	1712	1985	2218
Hudson/ACSS/TW	1158.4	13	0.9098	1.0281	2	25	7 x 0.1467	0.4401	1.196	1087	400.6	1488	31,100	33,500	38,800	0.0754	0.0943	1124	1389	1766	2051	2295
Catawba/ACSS/TW	1272.0	5	0.9991	1.0505	3	30	7 x 0.0967	0.2901	1.203	1197	174.0	1371	18,700	20,000	22,300	0.0695	0.0900	1152	1423	1807	2098	2346
Nelson/ACSS/TW	1257.1	7	0.9874	1.0557	3	35	7 x 0.1115	0.3345	1.213	1184	231.4	1415	22,100	23,800	26,900	0.0702	0.0907	1150	1420	1804	2094	2342
Yukon/ACSS/TW	1233.6	13	0.9689	1.0925	3	38	19 x 0.0910	0.4550	1.245	1165	419.2	1584	33,200	36,300	41,900	0.0712	0.0914	1154	1425	1810	2101	2350
Truckee/ACSS/TW	1372.5	5	1.0780	1.1334	3	30	7 x 0.1004	0.3012	1.248	1292	187.6	1479	20,200	21,500	24,000	0.0644	0.0838	1206	1491	1896	2203	2466
Mackenzie/ACSS/TW	1359.7	7	1.0679	1.1418	3	36	7 x 0.1159	0.3477	1.259	1281	250.0	1531	23,900	25,700	29,000	0.0649	0.0842	1206	1490	1895	2202	2465
Thames/ACSS/TW	1334.6	13	1.0480	1.1809	3	39	19 x 0.0944	0.4720	1.290	1260	451.1	1711	35,800	39,100	45,100	0.0658	0.0847	1210	1495	1902	2209	2472
St. Croix/ACSS/TW	1467.8	5	1.1529	1.2124	3	33	7 x 0.1041	0.3123	1.292	1381	201.7	1583	21,600	23,100	25,800	0.0602	0.0787	1256	1554	1979	2302	2578
Miramichi/ACSS/TW	1455.3	7	1.1430	1.2222	3	36	7 x 0.1200	0.3600	1.302	1372	268.0	1640	25,600	27,100	30,700	0.0607	0.0790	1269	1573	2007	2338	2577

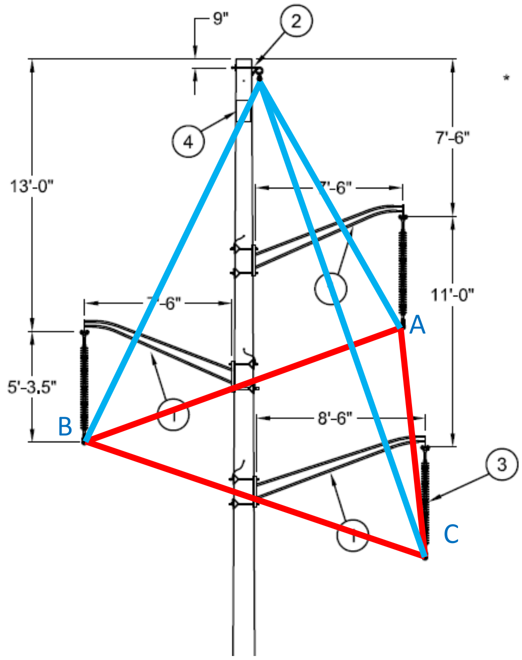
Southwire®   MADE IN THE USA

One Southwire Drive
Carrollton, GA 30119 USA
770/832-4242
www.southwire.com

Figure 1. Conductor information (ACSS/TW) produced by Southwire LLC.

DIMENSION AT A LINE POLE

The distances between conductors will be absolutely necessary for calculation because an electromagnetic field is one of the elements that can affect impedance and it is changed by distance among conductors.



Distance Aϕ to Bϕ

$= \sqrt{(7.5+7.5)^2 + (13-7.5)^2} = 15.977$ ft

Distance Bϕ to Cϕ

$= \sqrt{(7.5+8.5)^2 + (18.5-13)^2} = 16.919$ ft

Distance Cϕ to Aϕ

$= \sqrt{(8.5-7.5)^2 + 11^2} = 11.045$ ft

Distance Aϕ to Gϕ

$= \sqrt{7.5^2 + (7.5+5.29-0.75)^2} = 14.185$ ft

Distance Bϕ to Gϕ

$= \sqrt{7.5^2 + (13+5.29-0.75)^2} = 19.076$ ft

Distance Cϕ to Gϕ

$= \sqrt{8.5^2 + (18.5+5.29-0.75)^2} = 24.558$ ft

Figure 2. Pole Structure and Calculation of distance between conductors

The distances between conductors can be easily found through calculating the hypotenuse of a right-angled triangle similar to the example above which came from pole structure drawing.

GMR

GMR stands for 'Geometric Mean Radius'. The radius of Phase conductor is 1.108/2=0.554 as shown in Figure.1 table. Since each phase is not bundled and has the same kind of conductor, GMR is calculated as below:

$$GMR_a = GMR_b = GMR_c = Radius\ of\ Phasor\ Conductor \times e^{-\frac{1}{4}}\ in$$

$$= 0.554 \times e^{-1/4}\ in = 0.4315\ in = 0.036\ ft$$

Ground wire's radius is 0.2955 and GMR of ground wire is as follows:

$$GMR_g = 0.2955 \times e^{-1/4}\ in = 0.2301\ in = 0.0192\ ft$$

GMD

GMD stands for 'Geometric Mean Distance' and the method to get it is explained as below:

$$D_{ab} = 15.977\ ft$$
$$D_{bc} = 16.919\ ft$$
$$D_{ca} = 11.045\ ft$$

$$GMD = (D_{ab} \times D_{bc} \times D_{ca})^{1/3} = (15.977 \times 16.919 \times 11.045)^{1/3} = 14.339\ ft$$

CALCULATION OF LINE CONSTANTS (ASSUMED NO GROUND WIRE)

More commonly, real power systems include ground wire but this should be pre-comprehended as a basic concept of calculation self-impedance, and mutual impedance.

SELF-IMPEDANCE

$$Z_{aa} = Z_{bb} = Z_{cc} = (r_a + r_d) + j\omega k\, ln\frac{D_e}{GMR_a} = 0.1127 + 0.09528 + j0.12134 \times ln\frac{2790}{0.036}$$

$$= 0.20798 + j1.366 \;\; \Omega/mi$$

$$(D_e = 2160\sqrt{\frac{\rho}{f}} = 2160\sqrt{\frac{100}{60}} = 2790\; ft)$$

MUTUAL-IMPEDANCE

$$Z_{ab} = r_d + j\omega k\, ln\frac{D_e}{D_{ab}} = 0.09528 + j0.12134 \times ln\frac{2790}{15.977} = 0.09528 + j0.6264 \;\; \Omega/mi$$

$$Z_{bc} = r_d + j\omega k\, ln\frac{D_e}{D_{bc}} = 0.09528 + j0.12134 \times ln\frac{2790}{16.919} = 0.09528 + j0.6195 \;\; \Omega/mi$$

$$Z_{ac} = r_d + j\omega k\, ln\frac{D_e}{D_{ac}} = 0.09528 + j0.12134 \times ln\frac{2790}{11.045} = 0.09528 + j0.6712 \;\; \Omega/mi$$

INDUCTANCE

$$Inductance = 1.366\, \Omega/mi = 2.252 \times 10^{-6}\, H/m$$

CAPACITANCE

$$\overline{C_a} = \overline{C_b} = \overline{C_c} = \bar{C} = \frac{2\pi\epsilon}{ln(D_m/R_b)} = \frac{2\pi \times 8.854 \times 10^{-12}}{ln(14.399/0.036)} = 9.2852 \times 10^{-12}\, F/m$$

SUSCEPTANCE & SHUNT CAPACITIVE REACTANCE

$$\mathrm{B} = \mathrm{w}\bar{C} = 2\pi \times 60 \times 9.2852 \times 10^{-12} \; = 3.5 \times 10^{-9} mho/m = 5.6322 \times 10^{-6}\, mho/mi$$

SEQUENCE IMPEDANCE

$$Z_{abc} = \begin{bmatrix} 0.20798 + j1.366 & 0.09528 + j0.6264 & 0.09528 + j0.6712 \\ 0.09528 + j0.6264 & 0.20798 + j1.3669 & 0.09528 + j0.6195 \\ 0.09528 + j0.6712 & 0.09528 + j0.6195 & 0.20798 + j1.366 \end{bmatrix} \;\Omega/mi$$

$$Z_{012} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha \end{bmatrix}\begin{bmatrix} Z_{aa} & Z_{ab} & Z_{ac} \\ Z_{ab} & Z_{bb} & Z_{bc} \\ Z_{ac} & Z_{bc} & Z_{cc} \end{bmatrix}\begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha^2 & \alpha \\ 1 & \alpha & \alpha^2 \end{bmatrix}$$

CALCULATION OF LINE CONSTANTS (WITH GROUND WIRE)

If lines have ground wires, the total impedances of conductors would be changed as below because mutual impedances between phase conductors and ground wires should be considered.

$$Z_{ww} = (r_w + r_d) + j\omega k \ ln\frac{D_e}{GMR_g} = (0.7049 + 0.09528) + j0.12134 \times ln\frac{2790}{0.0192}$$

$$= 0.8002 + j1.4423 \ \ \Omega/mi$$

$$Z_{aw} = r_d + j\omega k ln\left(\frac{D_e}{D_{aw}}\right) = 0.09528 + j0.12134 \times \ln\frac{2790}{14.185} = 0.09528 + j0.6409 \ \ \Omega/mi$$

$$Z_{bw} = r_d + j\omega k ln\left(\frac{D_e}{D_{bw}}\right) = 0.09528 + j0.12134 \times ln\frac{2790}{19.076} = 0.09528 + j0.6049 \ \ \Omega/mi$$

$$Z_{cw} = r_d + j\omega k ln\left(\frac{D_e}{D_{cw}}\right) = 0.09528 + j0.12134 \times ln\frac{2790}{24.558} = 0.09528 + j0.5743 \ \ \Omega/mi$$

$$Z_{abc} = \begin{bmatrix} Z_{aa} - \frac{Z_{aw}Z_{aw}}{Z_{ww}} & Z_{ab} - \frac{Z_{aw}Z_{bw}}{Z_{ww}} & Z_{ac} - \frac{Z_{aw}Z_{cw}}{Z_{ww}} \\ Z_{ab} - \frac{Z_{aw}Z_{bw}}{Z_{ww}} & Z_{bb} - \frac{Z_{bw}Z_{bw}}{Z_{ww}} & Z_{bc} - \frac{Z_{bw}Z_{cw}}{Z_{ww}} \\ Z_{ac} - \frac{Z_{aw}Z_{cw}}{Z_{ww}} & Z_{bc} - \frac{Z_{bw}Z_{cw}}{Z_{ww}} & Z_{cc} - \frac{Z_{cw}Z_{cw}}{Z_{ww}} \end{bmatrix} \Omega/mi$$

$$Z_{abc} = \begin{bmatrix} 0.2614 + j1.1171 & 0.1437 + j0.3908 & 0.1395 + j0.4468 \\ 0.1437 + j0.3908 & 0.2518 + j1.1429 & 0.1352 + j0.4071 \\ 0.1395 + j0.4468 & 0.1352 + j0.4071 & 0.2443 + j1.1638 \end{bmatrix} \ \Omega/mi$$

LINE CONSTANT PROGRAM COMPARISON

The result of the example calculation above can be checked by comparing it to the result from the line constant program. In order to make this comparison, CAPE program was used. In order to run this program, the actual line length should be inserted and this amount is 3.64 miles. The results between the hand calculation and the CAPE line constant program are almost identical.

ACTUAL LINE IMPEDANCE BY HAND CALCULATION

$$Z_{abc} = \begin{bmatrix} 0.2614 + j1.1171 & 0.1437 + j0.3908 & 0.1395 + j0.4468 \\ 0.1437 + j0.3908 & 0.2518 + j1.1429 & 0.1352 + j0.4071 \\ 0.1395 + j0.4468 & 0.1352 + j0.4071 & 0.2443 + j1.1638 \end{bmatrix} \frac{\Omega}{mi} \times 3.64 \ mi$$

$$= \begin{bmatrix} 0.9515 + j4.0662 & 0.5231 + j1.4225 & 0.5078 + j1.6264 \\ 0.5231 + j1.4225 & 0.9166 + j4.1602 & 0.4921 + j1.4818 \\ 0.5078 + j1.6264 & 0.4921 + j1.4818 & 0.8893 + j4.2362 \end{bmatrix} \Omega$$

THE RESULT FROM CAPE PROGRAM

$$Z_{abc} = \begin{bmatrix} 0.9487 + j4.0613 & 0.5204 + j1.4324 & 0.5050 + j1.6360 \\ 0.5204 + j1.4324 & 0.9140 + j4.1541 & 0.4896 + j1.4908 \\ 0.5050 + j1.6360 & 0.4896 + j1.4908 & 0.8867 + j4.2293 \end{bmatrix} \Omega$$

About Authors

Seunghwa Lee holds a Bachelor's of Science degree in Electrical Engineering from Seoul National University, in South Korea. He also received a Masters of Engineering degree in Electrical Engineering from Texas A&M University with concentration in Power Systems. In 2014, Seunghwa became a registered professional engineer in the State of Texas (License No.118682). He is currently working on all projects related to power protection such as: protection design, relay setting, and forensic analysis of fault event records with proficiency in ASPEN and CAPE simulator.

Joe Perez holds a Bachelor's of Science degree in Electrical Engineering from Texas A&M University and is a registered professional electrical engineer in the state of Texas. He established SynchroGrid to provide electric utilities with power system protection applications, analysis, and training. Joe has provided services to relay manufacturers, utilities, and oil and gas companies. He has in-depth experience in power system protection settings and design from the manufacturer and utility perspectives. He has written papers for major relay conferences in the U.S. and many application notes and testing procedures for relay manufacturers and utilities. He is also the co-creator of a transformer rating software.

About SynchroGrid

SynchroGrid is a Texas-based company that offers electric utilities with professional consulting services, including transmission line, substation, and system protection design, relay settings development, relay coordination studies, and relay mis-operation analyses.

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